

# Effect of $N/Z$ equilibration on multinucleon-transfer yields in $^{48}\text{Ca} + ^{197}\text{Au}$

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## Abstract

We study multinucleon transfer in the  $^{48}\text{Ca} + ^{197}\text{Au}$  reaction at a beam energy of 400 MeV within the dinuclear system model. Nucleon exchange and competing DNS separation are described by a master equation with transition rates determined by the level densities of the dinuclear configurations. Calculations with and without imposed  $N/Z$  equilibrium give similar overall charge distributions, although differences remain in individual channels. When compared with the experimentally extracted DIC-like charge component, both treatments overpopulate the region near  $Z = 20$  and underpopulate more distant transfer channels. The calculated primary-fragment yields are concentrated near the entrance-channel partitions, with a pronounced maximum at  $Z = 20$  in the  $N = 28$  isotonic distribution. For the system and parameters considered, the overall charge distribution is only weakly sensitive to the  $N/Z$  equilibrium assumption. Isotope-resolved comparisons and a treatment of secondary decay are needed to assess the accuracy of the predicted final-fragment yields.

# 1 Introduction

Multinucleon transfer (MNT) reactions in deep-inelastic heavy-ion collisions offer a promising route to neutron-rich nuclei [1]. A theoretical description of these reactions requires a treatment of nucleon exchange between the interacting nuclei and their subsequent separation.

The dinuclear-system (DNS) description of deep-inelastic transfer reactions was introduced by Volkov [2,3] and later generalized to binary reaction dynamics without requiring complete dissipation of the relative kinetic energy [4].

In the present work, we apply the DNS model to the  $^{48}\text{Ca} + ^{197}\text{Au}$  reaction at a beam energy of 400 MeV. We compare the calculated charge distribution with experimental data obtained at the Flerov Laboratory of Nuclear Reactions of the Joint Institute for Nuclear Research [5]. The DNS evolution is described by a master equation that accounts for nucleon transfer and includes a loss term due to DNS separation (denoted by  $\Lambda_{\text{qf}}$  below for consistency with the standard DNS notation). The competition between these processes determines the yields of excited primary fragments. This treatment allows us to follow nucleon exchange until the DNS separates, without introducing a prescribed effective interaction time. A related DNS approach has also been used to describe nuclear fission in the fermium region [6].

Here we test whether imposing  $N/Z$  equilibrium materially changes the calculated primary-fragment yields and, in particular, whether the charge-integrated distribution  $Y(Z)$  can discriminate between the two descriptions. Isotopic and isotonic distributions are then used to identify information that is lost after summation over neutron number and to examine the possible role of shell structure.

# 2 Theoretical framework

We describe MNT reactions within a binary mechanism [4]. The colliding nuclei lose part of their relative kinetic energy and can become trapped in the potential pocket near contact, thereby forming a dinuclear system (DNS) in which the two nuclei remain identifiable while exchanging nucleons and energy. The fragments are treated as axially symmetric ellipsoids near contact, characterized by their charges  $Z_i$ , mass numbers  $A_i$ , and quadrupole deformations  $\beta_{2i}$  ( $i = 1, 2$ ) [7]. The DNS subsequently separates into two excited primary fragments, which may undergo secondary particle emission. The present calculation concerns the primary-fragment distributions.

The nucleus–nucleus interaction is written as

$$V_{\text{int}}(d) = V_C(d) + V_N(d), \quad (1)$$

where  $d$  is the distance between the nuclear centers. The Coulomb term includes quadrupole deformations, with ground-state binding energies and deformations taken from FRDM(2012) [10]; the nuclear term is evaluated by double folding the fragment densities with the density-dependent zero-range Migdal interaction [8, 11].

Figure 1 illustrates the resulting nucleus–nucleus potential for the  $^{48}\text{Ca} + ^{197}\text{Au}$  system. The competition between the attractive nuclear interaction and Coulomb repulsion produces a pocket near contact and an outer barrier against DNS separation. Trapping in this pocket provides the physical basis for formation of a

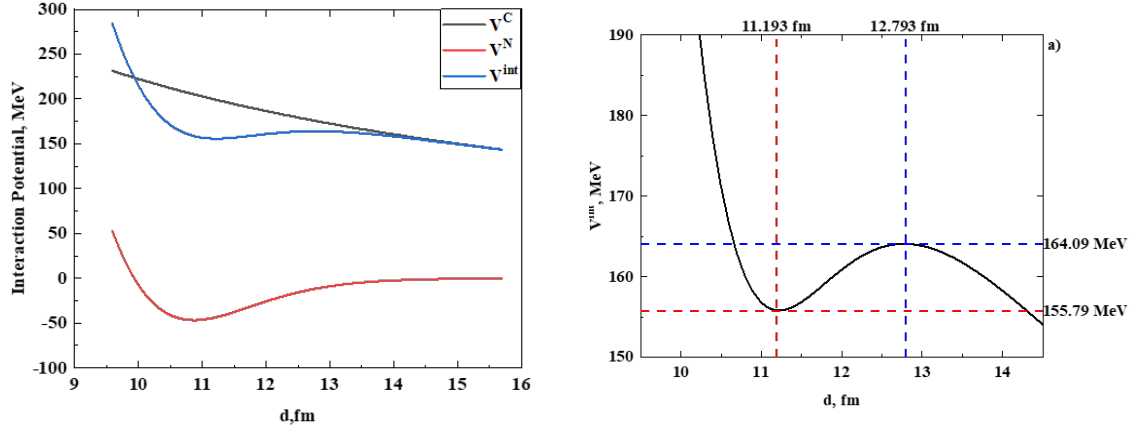


Figure 1: Nucleus–nucleus interaction potential for the  $^{48}\text{Ca} + ^{197}\text{Au}$  system. Left panel: Coulomb, nuclear, and total contributions. Right panel: enlarged view of the potential pocket near contact and the outer barrier against DNS separation for the representative deformation/orientation used in the calculation.

finite-lived DNS and allows nucleon exchange to proceed before separation. The outer barrier, shown more clearly in the right panel, determines the barrier entering the DNS-separation rate introduced below.

In the present calculation the angular-momentum dependence of the DNS potential-energy surface is neglected and the mass–charge landscape is evaluated at  $J = 0$ . Since the purpose of this work is a relative comparison of two transport treatments using the same potential-energy landscape, this approximation is expected to affect both calculations similarly. A partial-wave-dependent treatment is required for a quantitative description of absolute cross sections.

The intrinsic excitation energy assigned to a DNS configuration is

$$E^* = E_0^* + Q - V_{\text{int}}, \quad (2)$$

where  $E_0^* = E_0 - Q_{\text{initial}}$  denotes the entrance-channel excitation energy,  $E_0$  is the beam energy in the center-of-mass frame, and  $Q = E_b^{(1)} + E_b^{(2)} - E_b^{(c)}$ . Here  $E_b > 0$  denotes the positive ground-state nuclear binding energy.

For a transition from the entrance channel  $1 + 2$  to a configuration  $3 + 4$ , the corresponding energy balance can also be expressed as

$$U_f = U_i + Q_{gg} + \Delta V, \quad Q_{gg} = (M_1 + M_2 - M_3 - M_4)c^2, \quad \Delta V = V_{\text{int}}^{(i)} - V_{\text{int}}^{(f)}, \quad (3)$$

where  $M_i$  are ground-state masses.

In the statistical-equilibrium limit, channel populations follow the  $Q_{gg}$  systematics,

$$\sigma \propto \exp\left(\frac{Q_{gg} + \Delta E_C}{T}\right), \quad \Delta E_C = V_C^{(i)} - V_C^{(f)}, \quad (4)$$

where  $T$  is an effective temperature. This expression provides a useful equilibrium reference. In a DNS of finite lifetime, however, nucleon exchange competes continuously with separation, and the system may separate before this limit is reached. The time-dependent master equation therefore describes the non-equilibrium evolution explicitly rather than imposing statistical equilibration.

In the numerical calculation, the mass coordinate is discretized in unit steps,  $\Delta A = 1$ . Each configuration is labeled by  $n = (Z_1, A_1; Z_2, A_2)$ , with total charge and mass conserved, and neighboring cells are connected by one-nucleon transfer. Neutron and proton transfers are treated separately through the corresponding transition rates.

Let  $P(n, t)$  be the probability of occupying configuration  $n$  at time  $t$ . Its evolution is governed by the master equation [12],

$$\frac{dP(n, t)}{dt} = \sum_{n'} [\Lambda_{n' \rightarrow n} P(n', t) - \Lambda_{n \rightarrow n'} P(n, t)] - \Lambda_{\text{qf}}(n) P(n, t), \quad (5)$$

where  $\Lambda_{n \rightarrow n'}$  is the one-nucleon transfer rate and  $\Lambda_{\text{qf}}(n)$  is the loss rate along the relative-distance coordinate.

The two transport treatments compared below differ only in the allowed configuration space. In the unrestricted calculation, neutron and proton numbers can evolve independently through the neighboring one-nucleon-transfer channels. In the imposed-  $N/Z$  -equilibrium calculation, the evolution is constrained to the minimum-energy neutron configuration at each charge partition, thereby reducing the transport problem effectively to one dimension in  $Z$ .

The transfer rates are determined by microscopic transition strengths and by the intrinsic level densities of the connected configurations [8]. With the normalization used here,

$$\Lambda_{n \rightarrow n'} = 5\nu_0 \sqrt{\frac{\rho(E_{n'}, n')}{\rho(E_n^*, n)}}, \quad (6)$$

so that

$$\frac{\Lambda_{n \rightarrow n'}}{\Lambda_{n' \rightarrow n}} = \frac{\rho(E_{n'}, n')}{\rho(E_n^*, n)}. \quad (7)$$

Thus, the transport favors configurations with the larger available intrinsic phase space.

The DNS-separation loss rate is written as

$$\Lambda_{\text{qf}}(n) = \begin{cases} \nu_0 \sqrt{\frac{\rho(E_n^* - V_B(n), n)}{\rho(E_n^*, n)}}, & E_n^* > V_B(n), \\ 0, & E_n^* \leq V_B(n), \end{cases} \quad (8)$$

where  $V_B(n)$  is the outer barrier against separation from the DNS pocket; the corresponding barrier is illustrated for the representative  $^{48}\text{Ca} + ^{197}\text{Au}$  configuration in the right panel of Fig. 1. We retain the notation  $\Lambda_{\text{qf}}$  for continuity with the standard DNS formulation, although in the present MNT/DIC context it is most naturally interpreted as the DNS-separation loss rate.

The neutron- and proton-transfer strengths are taken equal,  $\lambda_n = \lambda_p \equiv \lambda_{\text{tr}} = 5\nu_0$ , while the reference separation strength is  $\lambda_{\text{qf}}^{(0)} = \nu_0$ . Thus the parameter controlling the competition between transfer and separation is the dimensionless ratio  $\lambda_{\text{tr}}/\lambda_{\text{qf}}^{(0)} = 5$ . This value was fixed using the  $^{51}\text{V} + ^{197}\text{Au}$  benchmark by comparison with the charge-distribution results discussed in Ref. [9].

The absolute value of the common scale  $\nu_0$  is not required for the asymptotic fragment-yield calculation: introducing  $\tau = \nu_0 t$  removes  $\nu_0$  from Eq. (5). The final

normalized yields therefore depend on the relative transfer and separation rates rather than on their common normalization.

Starting from  $P(n, 0) = P_0(n)$ , the master equation is solved by a Monte-Carlo procedure until the remaining DNS population has decayed. Equivalently, the final primary-fragment probability associated with configuration  $n$  can be viewed as the accumulated probability flux through the separation sink,

$$P_f(n) = \int_0^\infty \Lambda_{\text{qt}}(n) P(n, t) dt. \quad (9)$$

This makes explicit that the loss term in Eq. (5) generates the primary-fragment distribution rather than removing probability from the calculated yield.

The mass and charge distributions are

$$Y(A_1) = \sum_{Z_1} P_f(Z_1, A_1; Z_{\text{tot}} - Z_1, A_{\text{tot}} - A_1), \quad (10)$$

$$Y(Z_1) = \sum_{A_1} P_f(Z_1, A_1; Z_{\text{tot}} - Z_1, A_{\text{tot}} - A_1). \quad (11)$$

For comparison with the experimentally extracted DIC component, the normalized primary-fragment yields are converted to cross sections using a common normalization factor,

$$\sigma_{\text{calc}}^{(\text{DIC})}(Z) = \sigma_0 Y(Z), \quad \sigma_0 = 250 \text{ mb}. \quad (12)$$

The value of  $\sigma_0$  is estimated from the effective angular-momentum window contributing to the binary reaction, following the approximation of Ref. [4]. The same value is used for the calculations with and without imposed  $N/Z$  equilibrium. Since the principal aim of the present work is the comparison of the relative fragment-yield distributions, the conclusions are governed mainly by the relative population of the different transfer channels rather than by the absolute normalization.

For two fragments that retain their individual structure, the intrinsic DNS level density is approximated by the convolution

$$\rho_{\text{DNS}}(U) = \int_0^U \rho_1(E) \rho_2(U - E) dE, \quad (13)$$

[13]. For the numerical calculation, the fragment level densities are taken in the Fermi-gas form of Ref. [14], with  $a_i = A_i/12 \text{ MeV}^{-1}$  and the standard even-odd pairing shifts used there. The analytic convolution is applied in the regime  $\tilde{U} = U - \Delta_1 - \Delta_2 > 0$ .

### 3 Results and discussion

We apply the model to the  $^{48}\text{Ca} + ^{197}\text{Au}$  reaction at a beam energy of 400 MeV and compare the calculated charge distribution with the experimentally extracted DIC-like component of Ref. [5]. We examine how the assumption of  $N/Z$  equilibrium affects fragment yields when nucleon transfer competes with DNS separation.

Figure 2 compares calculations with and without imposed  $N/Z$  equilibrium. The two calculations give similar overall distributions, with local differences, particularly near  $Z = 22$ . For the present system and parameter set, the overall charge

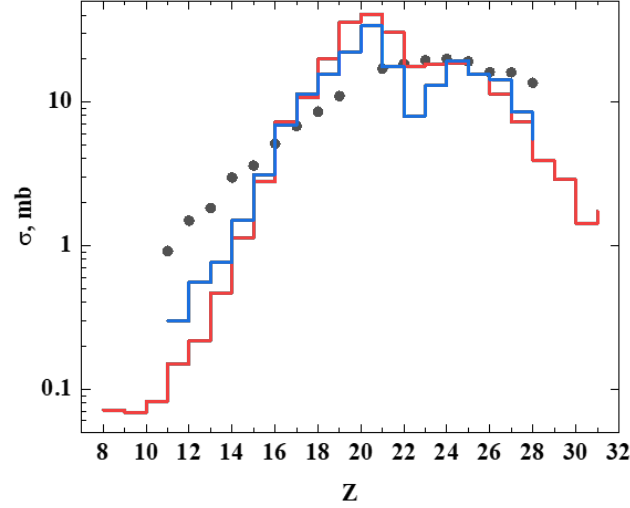


Figure 2: Charge distribution for the  $^{48}\text{Ca} + ^{197}\text{Au}$  reaction at 400 MeV. Points show the DIC-like component extracted from the experimental projectile-like-fragment yields of Ref. [5]. The red and blue curves show calculations without and with imposed  $N/Z$  equilibrium, respectively. The calculated curves correspond to excited primary fragments at DNS separation, whereas the experimental DIC component is measured after prompt de-excitation.

distribution is therefore only weakly sensitive to the imposed equilibrium condition. This similarity does not establish that  $N/Z$  equilibrium is reached: summation over neutron number may reduce differences between the two descriptions.

With the common normalization used in Fig. 2, both treatments overpopulate the region near  $Z = 20$  and underpopulate more distant transfer channels. The comparison is made specifically with the experimentally separated DIC-like component, so the principal reaction mechanism is consistent with the DNS description. Nevertheless, the calculation gives excited primary fragments at DNS separation, whereas the measured DIC products have already undergone prompt de-excitation. Neutron evaporation leaves  $Z$  unchanged and therefore makes the charge distribution more robust than isotope-resolved yields, while charged-particle emission can still redistribute strength between neighboring charge channels. Figure 2 should therefore be interpreted primarily as a comparison of the broad DIC charge pattern; a precision final-yield calculation requires secondary decay.

The similarity of the two calculated charge distributions is not sufficient to establish  $N/Z$  equilibration. Different isotope populations can give nearly the same

$$Y(Z) = \sum_N Y(Z, N),$$

so the charge-integrated yield may hide substantial differences in neutron transfer. The common discrepancy pattern motivates a sensitivity test of the relative transfer and separation rates, but does not by itself identify which rate should be changed.

Figure 3 illustrates the isotope-resolved structure that is lost when the yields are summed over neutron number. The positions and widths of the primary isotope distributions reflect both the available excitation energy and the transfer paths that can be reached before DNS separation; consequently, a maximum should not be interpreted as a direct measure of the stability of an isolated nucleus. Secondary

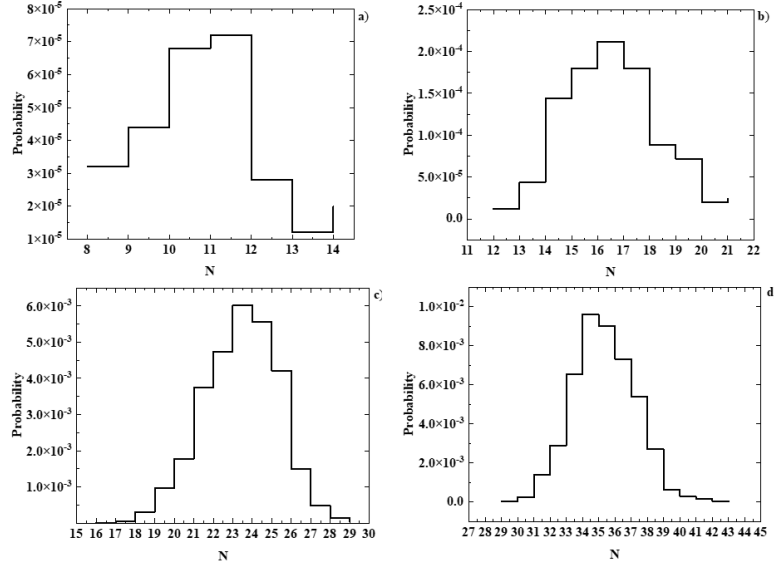


Figure 3: Calculated primary-fragment isotope distributions for (a) O ( $Z = 8$ ), (b) Mg ( $Z = 12$ ), (c) S ( $Z = 16$ ), and (d) Fe ( $Z = 26$ ) in the  $^{48}\text{Ca} + ^{197}\text{Au}$  reaction at 400 MeV.

decay can shift these distributions substantially. A direct isotope-resolved comparison of calculations with and without imposed  $N/Z$  equilibrium would provide a more discriminating test of equilibration than the charge-integrated distribution and remains a useful extension of the present analysis.

Figure 4 shows that the primary charge distribution remains concentrated near the entrance-channel partitions, with complementary peaks near  $Z = 20$  and  $Z = 79$  linked by charge conservation. Shell structure enters the present calculation mainly through the ground-state masses and deformations that determine the potential-energy landscape, whereas the intrinsic level densities are described by the comparatively smooth Fermi-gas expression of Sec. 2 with pairing shifts but without a separate microscopic shell-dependent level-density correction.

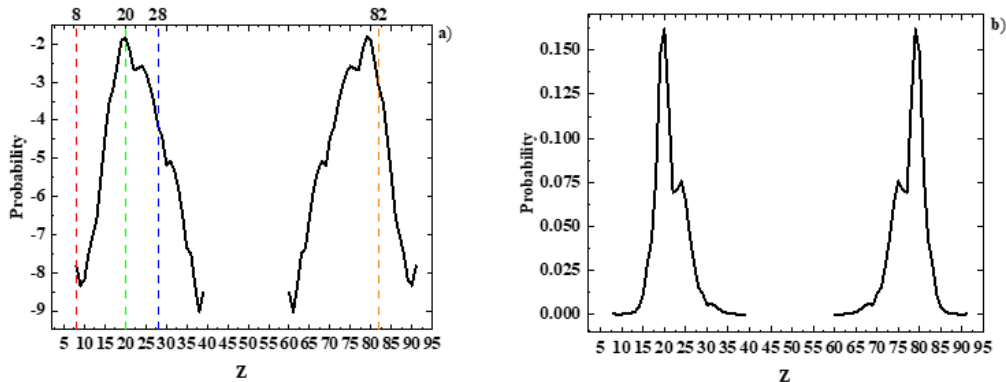


Figure 4: Calculated primary-fragment distributions for the  $^{48}\text{Ca} + ^{197}\text{Au}$  reaction at 400 MeV. (a) Full charge distribution shown as  $\ln Y(Z)$ ; vertical dashed lines mark  $Z = 8, 20, 28$ , and  $82$ . (b) Isotonic distribution for  $N = 28$ , with a maximum at  $Z = 20$ .

The  $N = 28$  isotonic distribution has a pronounced maximum at  $Z = 20$ , i.e. at doubly magic  $^{48}\text{Ca}$ , and this is consistent with a shell-favored energetic configuration. However,  $^{48}\text{Ca}$  is simultaneously the entrance-channel projectile, and the final yield also reflects accessible transfer paths and the finite DNS lifetime. The present calculation therefore does not provide an independent quantitative measure of shell enhancement and cannot separate the shell contribution from the entrance-channel bias.

Taken together, the calculations indicate that the charge-integrated yield  $Y(Z)$  is a comparatively insensitive observable for diagnosing  $N/Z$  equilibration in this system; isotope-resolved yields, together with secondary decay, are required for a more discriminating test.

## 4 Conclusions

We studied the  $^{48}\text{Ca} + ^{197}\text{Au}$  reaction at 400 MeV within the DNS model, treating one-nucleon exchange and DNS separation with a master equation. Calculations with and without imposed  $N/Z$  equilibrium produce similar overall charge distributions. When compared with the experimentally extracted DIC-like component, both calculations overpopulate the region near  $Z = 20$  and underpopulate more distant transfer channels. Thus, for the present system and parameter set, the charge-integrated distribution alone does not provide a sensitive test of  $N/Z$  equilibration.

The calculated primary yields remain concentrated near the entrance-channel partitions, including the  $Z = 20$ ,  $N = 28$  configuration. Because  $^{48}\text{Ca}$  is simultaneously doubly magic and the entrance-channel projectile, and because shell structure is included mainly through the potential-energy landscape rather than through a microscopic shell-dependent level density, the corresponding maximum cannot be interpreted as an independent quantitative shell enhancement. A more stringent assessment requires isotope-resolved comparison of the two transport treatments and secondary-decay calculations before confrontation with final isotope yields.

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