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Description of the decay $\phi \to \gamma\pi\pi$ in the quark NJL model

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Abstract

The decay $\phi \to \gamma\pi\pi$ is described within the $U(3) \times U(3)$ chiral Nambu–Jona-Lasinio model. It is shown that the inclusion of intermediate scalar mesons $f_0(500)$ and $f_0(980)$ plays an important role. All mesons are considered as quark-antiquark states.

1 Introduction

The study of the structure of light mesons and their interaction processes has a rich history. From a theoretical point of view, well-known methods for studying these states are phenomenological models based on the symmetry principles of strong interactions at low energies: chiral quark models and chiral perturbation theory, as well as extended versions of these models [1–10]. Quark models describe mesons of the scalar, pseudoscalar, vector, and axial-vector nonets. Among these states, the properties of scalar mesons are still insufficiently studied and are the subject of active discussion. This especially concerns the low-lying states $f_0(500)$, $f_0(980)$ and $a_0(980)$, where perturbative QCD is not applicable.

It should be noted that scalar mesons are key elements for the existence of chiral symmetry of strong interactions and its spontaneous breaking. The realization of chiral symmetry is achieved when considering scalar and pseudoscalar mesons with equal coupling constants. In the quark model, these mesons are considered as quark-antiquark systems. However, there is a large number of works in the literature describing scalar mesons as tetraquarks, meson molecules, and mixtures of these states (see the review papers, for example, [11–13]). Nevertheless, assuming that these states exist only in scalar mesons raises the question of preserving the chiral symmetry of strong interactions at low energies.

From an experimental point of view, the properties of light scalar mesons were studied in e^+e^- and $p\bar{p}$ annihilation processes [14–18], decays of heavy hadrons, and processes of meson beam interactions with nucleons [19–23]. In the present work, the radiative decay $\phi \to \gamma\pi^0\pi^0$ is considered. The study of such radiative decays of light vector mesons can help elucidate the nature of the scalar mesons $f_0(500)$ and $f_0(980)$, which participate as intermediate states. Experimentally, this process was studied at the CMD-2 detector at the VEPP-2M accelerator in Novosibirsk [14]. However, in the experimental study of this process, questions arise related to the representation of these states as quark or molecular systems. This circumstance gives particular importance to theoretical studies of such processes. In the present work, the process $\phi \to \gamma\pi\pi$ is considered within the framework of the quark Nambu–Jona-Lasinio (NJL) model [24–34]. This is a chiral quark phenomenological model that has successfully described a large number of meson interaction processes at low energies [34, 35]. In describing this decay, it will be shown that the inclusion of channels with intermediate scalar mesons $f_0(500)$ and $f_0(980)$ plays an important role. As shown in earlier works [32, 36, 37], the quark-antiquark representation of these mesons within the $U(3) \times U(3)$ NJL model using the 't Hooft interaction makes it possible to describe the masses of the scalar nonets in satisfactory agreement with experimental data.

2 NJL model

The NJL model is based on the chiral symmetry of strong interactions that is partially broken by the current masses of the u, d and s quarks. A detailed description of the model is given in review paper [26, 30–34] and references therein. The fragment of the quark-meson interaction Lagrangian containing the vertices necessary for the considered process has the following form [26, 34]:

$$\begin{aligned}\Delta L_{int} \;=\; & \bar{q}\left\{ig_\pi\gamma^5\lambda_0^\pi\pi^0 + g_\phi\gamma^\mu\lambda^\phi\phi_\mu + \left(g_\sigma\lambda_u^\sigma\cos\bar{\theta}_\sigma - g_\sigma^s\lambda_s^\sigma\sin\bar{\theta}_\sigma\right)\sigma\right.\\ & \left.+\left(g_\sigma\lambda_u^\sigma\sin\bar{\theta}_\sigma + g_\sigma^s\lambda_s^\sigma\cos\bar{\theta}_\sigma\right)f_0\right\}q, \qquad (1)\end{aligned}$$

where q is the quark triplet, $\phi = \phi(1020)$, $\sigma = f_0(500)$, $f_0 = f_0(980)$, $\bar{\theta}_\sigma = \theta_0 - \theta_\sigma$ is the mixing angle of the scalar mesons, $\theta_0 = 35.3°$ is the ideal mixing angle, $\theta_\sigma = 24°$, λ are linear combinations of the Gell-Mann matrices [34, 35].

The quark-meson coupling constants take the form

$$g_\pi = \sqrt{\frac{Z_\pi}{4I_{20}}}, \quad g_\sigma = \sqrt{\frac{1}{4I_{20}}}, \quad g_\sigma^s = \sqrt{\frac{1}{4I_{02}}}, \quad g_\phi = \sqrt{\frac{3}{2I_{02}}}\quad, \qquad (2)$$

where Z_π is an additional renormalization constant arising from the $a_1 - \pi$ transitions:

$$Z_\pi = \left(1 - 6\frac{m_u^2}{M_{a_1(1260)}^2}\right)^{-1}. \qquad (3)$$

Here $m_u = 270$ MeV is the constituent u-quark mass, and $M_{a_1(1260)} = 1230$ MeV is the mass of the $a_1(1260)$ meson [38].

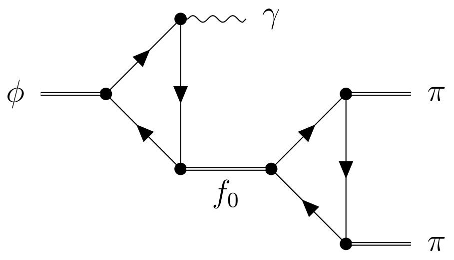

Figure 1: Quark diagram for the decay $\phi \to \gamma\pi\pi$, where $f_0 = f_0(500), f_0(980)$.

The integrals I_{nm} take the following form:

$$I_{nm} = -i\frac{N_c}{(2\pi)^4}\int \frac{\theta(\Lambda^2 + k^2)}{(m_u^2 - k^2)^n(m_s^2 - k^2)^m}\mathrm{d}^4k, \tag{4}$$

where $\Lambda = 1265$ MeV is the cutoff parameter [26].

3 Decay $\phi \to \pi\pi\gamma$

The decay diagram for $\phi \to \pi\pi\gamma$ is presented in Fig. 1.

The decay amplitude includes the vertices $\phi \to \gamma f_0$ and $f_0 \to \pi\pi$. The vertex $f_0 \to \pi\pi$ can be obtained in the standard way for the NJL model by expanding the quark loop in a series in powers of the external momenta and keeping only the divergent terms regularized by the cutoff:

$$\mathcal{L} = 4m_u Z_\pi g_{f_0}\left[f_0\pi^0\pi^0 + \frac{3}{M_{a_1}^2}\left(\partial_\mu\partial^\mu f_0\pi^0\pi^0 - f_0\pi^0\partial_\mu\partial^\mu\pi^0\right)\right], \tag{5}$$

where the second term in square brackets describes the $a_1 - \pi$ transitions at the pions in the final state, $g_{f_0} = g_\sigma\cos\bar{\theta}_\sigma$ for the case $f_0 = f_0(500)$, and $g_{f_0} = g_\sigma\sin\bar{\theta}_\sigma$ for the case $f_0 = f_0(980)$.

The amplitude of the decay $\phi \to \gamma f_0$ was obtained in [39]:

$$\mathcal{M}(\phi \to \gamma f_0) = \frac{e}{(4\pi)^2}g_\phi g_{f_0}^s Re\left(C_{f0}\right)\left[g^{\mu\nu}\left(p, p_\gamma\right) - p^\nu p_\gamma^\mu\right]e_\mu^*(p)e_\nu(p_\gamma), \tag{6}$$

where p is the momentum of the ϕ meson, p_γ is the photon momentum, $g_{f_0}^s = -g_\sigma^s\sin\bar{\theta}_\sigma$ for the case $f_0 = f_0(500)$, and $g_{f_0}^s = g_\sigma^s\cos\bar{\theta}_\sigma$ for the case $f_0 = f_0(980)$.

The convergent integral over the quark loop takes the following form:

$$\begin{aligned}
C_{f0} &= \int_0^1 dx \int_0^{1-x} dy \frac{8m_s(4xy-1)}{m_s^2 - y(1-y)M_\phi^2 + xy\left(M_\phi^2 - M_{f_0}^2\right)} = \frac{8}{m_s}\left(4C_1^{f0} - C_2^{f0}\right), \\
C_1^{f0} &= \frac{1}{2(a-b)} - \frac{2}{(a-b)^2}\left[f\left(\frac{1}{b}\right) - f\left(\frac{1}{a}\right)\right] + \frac{a}{(a-b)^2}\left[g\left(\frac{1}{b}\right) - g\left(\frac{1}{a}\right)\right], \\
C_2^{f0} &= \frac{2}{a-b}\left[f\left(\frac{1}{b}\right) - f\left(\frac{1}{a}\right)\right], \\
f(x) &= \begin{cases} -\left[\arcsin\left(\frac{1}{2\sqrt{x}}\right)\right]^2, & x > \frac{1}{4}, \\ \frac{1}{4}\left[\ln\left(\frac{\eta_+}{\eta_-}\right) - i\pi\right]^2, & x < \frac{1}{4} \end{cases}, \\
g(x) &= \begin{cases} \sqrt{4x-1}\arcsin\left(\frac{1}{2\sqrt{x}}\right), & x > \frac{1}{4}, \\ \frac{1}{2}\sqrt{1-4x}\left[\ln\left(\frac{\eta_+}{\eta_-}\right) - i\pi\right], & x < \frac{1}{4} \end{cases}, \\
\eta_\pm &= \frac{1}{2}\left(1 \pm \sqrt{1-4x}\right), \quad a = \frac{M_\phi^2}{m_s^2}, \quad b = \frac{M_{f_0}^2}{m_s^2}.
\end{aligned} \tag{7}$$

Then the amplitude of the process $\phi \to \pi\pi\gamma$ is represented as

$$\begin{aligned}
\mathcal{M}(\phi \to \pi\pi\gamma) &= -4\frac{e}{(4\pi)^2} g_\phi g_\sigma^s m_u Z_\pi g_\sigma \sin\bar{\theta}_\sigma \cos\bar{\theta}_\sigma \\
&\times \left\{C_{f_0(500)} BW_{f_0(500)}(q) - C_{f_0(980)} BW_{f_0(980)}(q)\right\} \\
&\times \left[g^{\mu\nu}\left(p, p_\gamma\right) - p^\nu p_\gamma^\mu\right] e_\mu^*(p) e_\nu(p_\gamma),
\end{aligned} \tag{8}$$

where the intermediate states are described by the Breit-Wigner propagator

$$BW(q^2) = \frac{1}{M_{f_0}^2 - q^2 - i\sqrt{q^2}\Gamma_{f_0}}, \tag{9}$$

where M_{f_0} and Γ_{f_0} are meson mass and width [38]. The experimental value of the width of this decay is

$$\Gamma(\phi \to \pi\pi\gamma)_{exp} = 480 \pm 25 \text{ eV}. \tag{10}$$

The experimental values of the widths of the scalar mesons participating in this process as intermediate states have a wide range of possible values. We present results for two combinations of the total widths of the intermediate states. Perfect agreement with experiment occurs in the case $\Gamma_{f0(500)} = 800$ MeV, $\Gamma_{f0(980)} =$ 33 MeV:

$$\Gamma(\phi \to \pi\pi\gamma) = 480 \pm 72 \text{ eV}. \tag{11}$$

In the case $\Gamma_{f0(500)} = 600$ MeV, $\Gamma_{f0(980)} = 33$ MeV we obtain

$$\Gamma(\phi \to \pi\pi\gamma) = 413 \pm 62 \text{ eV}, \tag{12}$$

which, within the theoretical and experimental errors, is also consistent with the experiment. The theoretical error of the model is estimated at level 15% [34, 35]. The main source affecting the accuracy of the model is the violation of the chiral symmetry, associated mainly with the non-zero current quark masses. Statistical analysis of numerous calculations shows that, in most cases, satisfactory agreement with experiment at low energies is achieved within the above-mentioned uncertainty.

4 Conclusion

The process $\phi \to \pi\pi\gamma$ considered in this paper is interesting, since its study contributes to a deeper understanding of the nature of scalar mesons. In the calculation of the decay $\phi \to \pi\pi\gamma$, experimentally known values of the masses and total widths of the $f_0(500)$ and $f_0(980)$ states are used. Within the experimentally allowed ranges for these quantities, and also taking into account the theoretical and experimental errors for the width of the decay $\phi \to \pi\pi\gamma$, results were obtained that are in satisfactory agreement with the experimental data. The calculations show that, along with various representations of the structure of scalar mesons (see the review articles, for example, [11–13]), the $\bar{q}q$ representation works quite satisfactorily at low energies, where scalar mesons are considered as chiral symmetric partners of pseudoscalar mesons. Theoretical estimates for the decay $\phi \to \pi^0\pi^0\gamma$ were also obtained within dispersion theory, where final-state pion interactions were considered [45]. In [46], the method of effective chiral Lagrangians was applied. Our results are in reasonable agreement with the results of these works.

In the NJL model, the masses of the scalar mesons are described in satisfactory agreement with experiment, except for the isovector state $a_0(980)$ [36, 40]. The theoretical estimate of the mass of this meson is $M_{a_0(980)} = 810$ MeV, while the experimental value is $M_{a_0(980)} = 980 \pm 20$ MeV [38]. A similar situation was observed in the description of the mass spectrum of the ground states of the scalar mesons together with their first radially excited states and the scalar glueball. As a result, agreement with experiment was achieved for 18 mesons, excluding the mass of the ground state of the scalar a_0 meson. For this meson mass, the value $M_{a_0(980)} = 830$ MeV was obtained [32, 37]. All this suggests that for the isoscalar $f_0(500)$, $f_0(980)$ and the strange scalar mesons $K_0^*(800)$, the representation in the form of $q\bar{q}$ systems is quite acceptable.

The recent results of the quark NJL model for processes involving scalar mesons should be noted. In [41], the decay $\tau \to 3\pi\nu_\tau$ was considered taking into account the intermediate channel with the scalar meson f_0. In that work, good results were obtained for the spectral function and the decay width, consistent with the CLEO data [42]. At the same time, the important role of the contribution of the $f_0\pi$ channel to the width of the decay $\tau \to 3\pi\nu_\tau$ was shown, which improved agreement with experiment. Within this approach, the strong decays of the strange scalar mesons $K_0^*(700)$ and $K_0^*(1430)$ have also been studied [43]. In addition, the production of strange scalar mesons in τ-lepton decays was considered in Ref. [44].

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