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Decays of strange scalar mesons $K_0^*(800)$ and $K_0^*(1430)$

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Abstract

In the NJL quark model, the elements of constructing a Lagrangian for strange scalar mesons based on the effective four-quark interaction, motivated by quantum chromodynamics are presented. The meson mass spectrum is calculated, and numerical estimates of the model parameters are given. The NJL Lagrangians are used to calculate the main strong decays of scalar mesons. Theoretical predictions for the production of scalar mesons by the τ lepton current are obtained.

1 Introduction

The Nambu – Jona-Lasinio (NJL) type quark models based on the chiral symmetry group $U(3)\times U(3)$ are an effective tool for studying the interactions of light mesons at low energies [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. In the quark model, mesons of scalar, pseudoscalar, vector, and axial-vector types are considered as bound quark-antiquark states. The model successfully describes the meson mass spectrum and the processes of their interaction on the low-energy scale. Nevertheless, when analyzing the scalar meson nonet, difficulties arise in explaining the mass of the isovector meson $a_0(980)$. Additionally, problems are observed in describing the decay widths $f_0(980) \to 2\pi$ and $a_0(980) \to \pi\eta$, where the first width according to the model turns out to be underestimated, while the second width exceeds the experimental data [13, 14]. To overcome these problems, a hypothesis is currently being introduced regarding the complex nature of the scalar $f_0(980)$ and $a_0(980)$ mesons, in which, along with traditional $\bar{q}q$ structures, kaonic molecular states or tetraquark configurations play a leading role [15, 16, 17]. It is important to note that the remaining states $f_0(500)$ and $K_0^*(800)$ from scalar nonet and their excitations are well described in the $\bar{q}q$ representation, which is critical for ensuring the chiral symmetry of strong interactions.

Of great interest is the study of both the ground and excited states of scalar mesons especially the strange resonances $K_0^*(800)$ and $K_0^*(1430)$ in the view of new experimental data. Recent measurements by the BaBar collaboration, indicating a significant discrepancy with the PDG data for the decay width of $K_0^*(1430) \to K\pi$ emphasize the relevance of theoretical studies of processes involving strange scalar mesons [18, 19]. In the BaBar study [18], estimates for the ratio $B(\eta_c \to \eta' K)/B(\eta_c \to \pi K)$ taking into account the contribution of channels with the resonance $K_0^*(1430)$ for the first time were obtained and values for the coupling constants $g_{\eta' K}$ and $g_{\pi K}$ were presented.

In this paper, the main decay channels of the strange scalar meson $K_0^*(800)$ in the ground and first excited states are considered. The matrix elements of the decays of the meson $K_0^*(1430)$ into the $K\pi$, $K\eta$, and $K\eta'$ channels are presented. In addition, theoretical estimates will be obtained for the branching fractions of τ decays with the production of scalar mesons $\tau^- \to \nu_\tau K_0^*(800)^- \pi^0 \nu_\tau$ $\tau \to K_0^*(1430)^- \pi^0 \nu_\tau$, $\tau^- \to K_0^*(700)^- K^0 \nu_\tau$ and $\tau \to K_0^*(700)^- \eta \nu_\tau$.

2 The effective Lagrangian of the NJL model for strange scalar mesons and the mass spectrum

One of the aim of the current paper is to confirm the hypothesis of a quark-antiquark structure of scalar strange mesons. One of the main arguments in favor of this hypothesis is the correct description of the masses of strange scalar mesons in the ground and first radially excited states. In addition, this paper the decays of scalar mesons and the processes of their production by the τ lepton current will be considered. The standard NJL model describes meson nonets and their interactions in the ground state in good agreement with data [1, 2, 3]. The standard quark NJL model is based on the effective four-quark interaction. The model describes the mechanism of spontaneous chiral symmetry breaking. The model was extended by including the first radially excited states of mesons. The extension was achieved by introducing a polynomial form factor into the initial effective four-quark interaction $f_i(k_\perp^2) = 1 + d_i k_\perp^2$, where k is the relative transverse momentum of quarks in a meson [8, 10, 11, 12]. Let us consider the extended $U(3)\times U(3)$ version of the model where the initial effective four-quark Lagrangian taking into account the 't Hooft interaction takes the form

$$\begin{aligned}
&\mathcal{L}(\bar{q},q) = \\
&\quad \int d^4x\ \bar{q}(x)(i - m_0)q(x) \\
&\quad + \frac{1}{2}\int d^4x \sum_{a=1}^{9}\sum_{b=1}^{9}[G_{ab}^{(-)} j_{S,1}^a(x) j_{S,1}^b(x) + G_{ab}^{(+)} j_{P,1}^a(x) j_{P,1}^b(x)] \\
&\quad + \frac{G_S}{2}\int d^4x \sum_{a=1}^{9}\sum_{i=2}^{N}[j_{S,i}^a(x) j_{S,i}^a(x) + j_{P,i}^a(x) j_{P,i}^a(x)] \\
&\quad - \frac{G_V}{2}\int d^4x \sum_{a=1}^{9}\sum_{i=1}^{N}[j_{V,i}^{a,\,\mu}(x) j_{V,i,\,\mu}^a(x) + j_{A,i}^{a,\,\mu}(x) j_{A,i,\,\mu}^a(x)], \qquad (1)
\end{aligned}$$

where G_S and G_V are the four-quark interaction constants for scalar (pseudoscalar) and vector (axial-vector) types; the definition of the constants $C_{ab}^{\pm}$ can be found in the work [8]; m_0 is the matrix of current quarks. $j_{U,i}$ with $U = S, P, V, A$ denotes the scalar, pseudoscalar, vector, and axial-vector quark currents

$$\begin{aligned}
j_{S(P),i}^a(x) &= \int d^4x_1 d^4x_2\ \bar{q}(x_1) F_{S(P),i}^a(x; x_1, x_2) q(x_2), \\
j_{V(A),i}^{a,\,\mu}(x) &= \int d^4x_1 d^4x_2\ \bar{q}(x_1) F_{V(A),i}^{a,\,\mu}(x; x_1, x_2) q(x_2), \qquad (2)
\end{aligned}$$

where $F_{S(P),i}^a(x; x_1, x_2)$ are the scalar (pseudoscalar) and $F_{V(A),i}^{a,\,\mu}(x; x_1, x_2)$ the vector and axial-vector form factor.

After the bosonization procedure in the one-loop quark approximation, we can obtain the following Lagrangian

$$
\begin{aligned}
&L_{\mathrm{bos}}(\sigma,\varphi,V,A) = \\
&-\sum_{a,b=1}^{9}\int d^4x\left[\frac{1}{2}\left(\left(G^{(-)}\right)^{-1}_{ab}\bar{\sigma}^a_1(x)\bar{\sigma}^b_1(x)+\left(G^{(+)}\right)^{-1}_{ab}\varphi^a_1(x)\varphi^b_1(x)\right)\right. \\
&\left.-\frac{1}{2G_V}\left((V^{a,\mu}_1(x))^2+(A^{a,\mu}_1(x))^2\right)\right] \\
&-\sum_{a=1}^{9}\int d^4x\left[\frac{1}{2G}\left((\sigma^a_2(x))^2+(\phi^a_2(x))^2\right)-\frac{1}{2G_V}\left((V^{a,\mu}_2(x))^2+(A^{a,\mu}_2(x))^2\right)\right] \\
&-i\mathrm{Tr}\ln\left[1+\frac{1}{i\not{\partial}-m}\sum_{j=1}^{2}\sum_{a=1}^{9}(\sigma^a_j+\varphi^a_j+V^{a,\mu}_j\gamma_\mu+A^{a,\mu}_j\gamma_5\gamma_\mu)f^a_j\tau_a\right]. \qquad (3)
\end{aligned}
$$

This Lagrangian describes scalar, pseudoscalar, vector, and axial-vector states. Now consider the part of the Lagrangian (3) for strange scalar mesons

$$
\begin{aligned}
\mathcal{L}(K^*_{0,2}) = &-\frac{{K^*_{0,1}}^2}{G_{K^*_0}}-\frac{1}{G}((K^*_{0,2})^2) \\
&-iN_cTr\ln\left[1+\frac{1}{i\partial_\mu\gamma_\mu-m}\sum_{a=4}^{7}\sum_{j=1}^{2}\tau_a\left[\sigma^a_j\right]f^a_j\right], \qquad (4)
\end{aligned}
$$

where scalar fields are $\sum_{a=4}^{7}(\sigma^a_j)^2 \equiv 2{K^*_{0,j}}^2 = 2(\bar{K}^*_{0,j})^0(K^*_{0,j})^0+2(K^*_{0,j})^+(K^*_{0,j})^-$.

The free Lagrangian for strange scalar mesons after renormalization has the form (details of the renormalization procedure for other states can be found in the review paper [8, 11, 12])

$$
\begin{aligned}
\mathcal{L}^{(2)}_{K^*_0} &= \frac{1}{2}\left(P^2-(m_u+m_s)^2-M^2_{K^*_0,1}\right)K^{*2}_{0,1}+\Gamma_{K^*_0}\left(P^2-(m_u+m_s)^2\right)K^*_{0,1}K^*_{0,2} \\
&+ \frac{1}{2}\left(P^2-(m_u+m_s)^2-M^2_{K^*_0,2}\right)K^{*2}_{0,2}, \qquad (5)
\end{aligned}
$$

where $\Gamma_{K^*_0}=\frac{I^{fa}_2}{\sqrt{I_2 I^{ffa}_2}}$. This Lagrangian has a non-diagonal form. Diagonalization is achieved by the following redefinition of the meson fields

$$
\begin{aligned}
\sigma^a &= \cos(\theta_{\sigma,a}-\theta^0_{\sigma,a})\sigma^{ar}_1-\cos(\theta_{\sigma,a}+\theta^0_{\sigma,a})\sigma^{ar}_2, \\
\hat{\sigma}^a &= \sin(\theta_{\sigma,a}-\theta^0_{\sigma,a})\sigma^{ar}_1-\sin(\theta_{\sigma,a}+\theta^0_{\sigma,a})\sigma^{ar}_2. \qquad (6)
\end{aligned}
$$

As a result, the free Lagrangian for strange scalar meson fields takes the standard diagonal form

$$
L^{(2)}_{K^*_0} = \frac{1}{2}(P^2-M^2_{K^*_0})\,K^{*2}_0+\frac{1}{2}(P^2-M^2_{\hat{K^*_0}})\hat{K}^{*2}_0. \qquad (7)
$$

The corresponding mass formulas for the states $M_0^*(800)$ and $M_0^*(1430)$ have the form [8]

$$M^2_{(K_0^*,K_0^{*\prime})} = \frac{1}{2(1-\Gamma^2_{K_0^*})}\Bigg[M^2_{K_0^*,1} + M^2_{K_0^*,2} \pm \sqrt{\left(M^2_{K_0^*,1} - M^2_{K_0^*,2}\right)^2 + \left(2M_{K_0^*,1}M_{K_0^*,2}\Gamma_{K_0^*}\right)^2}\Bigg] + (m_u+m_s)^2, \tag{8}$$

where $\Gamma_{K_0^*} = I_{11}^f/\sqrt{I_{11}I_{11}^{f^2}} = 0.49$. The masses of the non-physical mesons $M_{K_0^*,1}$ and $M_{K_0^*,2}$ are defined as follows

$$M^2_{K_0^*,1} = \frac{1}{4I_{11}}\left[\frac{1}{G_{K_0^*}} - 4\left(I_{10}+I_{01}\right)\right], \tag{9}$$

$$M^2_{K_0^*,2} = \frac{1}{4I_{11}^{f^2}}\left[\frac{1}{G_{K_0^*}} - 4\left(I_{10}^{f^2}+I_{01}^{f^2}\right)\right], \tag{10}$$

where $G_{K_0^*} = 3.88\times10^{-6}\mathrm{MeV}^{-2}$ is the effective four-quark interaction constant [8]. I_{10} and I_{01} are quadratically divergent integrals. There are six main parameters in extended NJL model: the masses of the constituent quarks $m_{u(d)}$ and m_s, the cutoff parameter Λ_3, the four quark interaction constants G_S and G_V, and the 't Hooft constant K. The mass of the constituent quark and the cutoff parameter are fixed by the decays widths of $\pi\to\mu\nu_\mu$ and $\rho\to2\pi$. The strange quark mass and the constants G_S, G_V and K are fixed from the masses of the charged pion, K meson, ρ meson, and mesons η/η'. The numerical values for model parameters are $m_u = 270$ MeV, $m_s = 420$ MeV, $\Lambda_3 = 1.03$ GeV, $G = 3.14$ GeV^{-2}, $G_V = 12$ GeV^{-2}, $K = 6.1$ GeV^{-5}.

As a result, for the masses of scalar mesons $M_0^*(700)$ and $M_0^*(1430)$ we obtain the following values:

$$M_{K_0^*(700)} = 940\,\mathrm{MeV}, \quad M_{K_0^*(1430)} = 1425\,\mathrm{MeV}. \tag{11}$$

The currently experimental values for meson mass are $M^{exp}_{K_0^*(800)} = 845\pm17\mathrm{MeV}$ and $M^{exp}_{K_0^*(1430)} = 1425\pm50\mathrm{MeV}$ [19]. At the same time, there are studies that give relatively higher values $M_{K_0^*(800)} = 905(+65,-30)\,\mathrm{MeV}$ [20].

Our results are in qualitative agreement with the experimental data. It is interesting to note that the obtained theoretical values for the masses show that strange scalar mesons mainly have a quark-antiquark structure. And here there is no need to introduce additional tetraquark structures, which would lead to a large value of the meson masses and a deviation from the experimentally known for the masses.

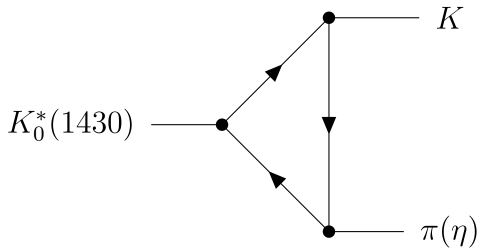

Figure 1: Triangle quark diagram for the decay of scalar meson K_0^* with production of meson pair $K\pi$, $K\eta$ and $K\eta'$

3 Decays of scalar mesons $K_0^*(800)$ and $K_0^*(1430)$

Two-particle decays of scalar mesons $K_0^*(800)$ and $K_0^*(1430)$ with meson pair $K\pi$, $K\eta$ and $K\eta'$ production are described by the quark diagrams shown in Fig.1. Calculations performed in the NJL model for the decay $K_0^{*-}(800) \to K^-\pi^0$ yield a following decay width [21]

$$\Gamma(K_0^{*-} \to K^-\pi^0) = \frac{\left(8m_s I_{11}^{K_0^* K\pi}\right)^2}{2M_{K_0^*}} \frac{\sqrt{E_K^2 - M_K^2}}{4\pi M_{K_0^*}}, \tag{12}$$

where E_K is the kaon energy in the rest system of K^*, the integral over the quark loop has the form

$$I_{n_1 n_2}^{MM'\dots}(m_u, m_s) = -i\frac{N_c}{(2\pi)^4}\int \frac{A(k_\perp^2)\dots B(k_\perp^2)\dots}{(m_u^2 - k^2)^{n_1}(m_s^2 - k^2)^{n_2}}\Theta(\Lambda_3^2 - \vec{k}^2)\mathrm{d}^4k, \tag{13}$$

where $A(k_\perp^2)$ and $B(k_\perp^2)$ are the coefficients for different mesons given in Ref. [21].

The resulting decay width with various meson charge distributions in the final state is $\Gamma(K_0^{*-} \to K^-\pi^0 + K^0\pi^-) = 430 \pm 64$ MeV [21]. Our results do not contradict with theoretical decay width $\Gamma(K_0^*(800) \to K\pi) = 401.1 \pm 87.1$ MeV obtained in [22]. The experimental value for the width of this decay is $\Gamma(K_0^* \to K\pi)_{exp} = 468 \pm 30$ MeV [19].

Decays of the radially excited scalar meson $K_0^*(1430) \to K\pi$ are described by a similar width (12) with the meson vertices replaced. The decay amplitudes with meson pair $K\eta$ and $K\eta'$ have the form

$$\mathcal{M}(K_0^*(1430) \to K\eta) = 8m_s I_{11}^{\hat{K}_0^* K\eta_u} - 8\sqrt{2}m_u I_{11}^{\hat{K}_0^* K\eta_s}, \tag{14}$$

$$\mathcal{M}(K_0^*(1430) \to K\eta') = 8m_s I_{11}^{\hat{K}_0^* K\eta'_u} - 8\sqrt{2}m_u I_{11}^{\hat{K}_0^* K\eta'_s}, \tag{15}$$

where loop integrals and used parameters can be found in [21].

Table 1: The comparison of the decay widths and constants with BaBar data [18].

Decay mode	Decay widths, MeV	[18]	Decay constant, GeV^2	[18]
$K_0^*(1430) \to K\pi$	18.54 ± 2.87	16.46 ± 1.15	$g^2_{K\pi} = 0.515$	$g^2_{K\pi} = 0.458$
$K_0^*(1430) \to K\eta$	0.29 ± 0.04	-	$g^2_{K\eta} = 0.03$	-
$K_0^*(1430) \to K\eta'$	-	-	$g^2_{K\eta'} = 0.67$	-

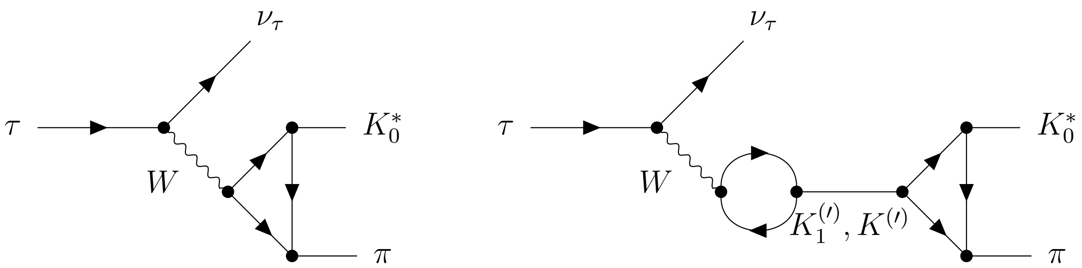

Figure 2: Diagrams contributing to the decay $\tau \to K_0^* \pi \nu_\tau$.

The results of the model calculations for the decays of $K_0^*(800, 1430)$ mesons are presented in Table 1.

The τ decay into meson pair $K_0^*(800)\pi$ with scalar meson is described by quark diagrams in the Fig. 2.

As a result, for the total decay amplitude $\tau \to K_0^*(800)\pi\nu_\tau$ we obtain [23]

$$\mathcal{M}(\tau \to K_0^*(800)\pi\nu_\tau) = 2G_F V_{us} L_\mu \Big[\mathcal{M}_C + \mathcal{M}_{K_1+K_1'} + \mathcal{M}_{K+K'}\Big]_{\mu\nu} \big(p_{K_0^*} - p_\pi\big)_\nu, \quad (16)$$

where L_μ is the lepton current, $p_{K_0^*}$ and p_π are the momenta of the scalar meson and pion. The contribution from the contact diagram is $\mathcal{M}_{C\mu\nu} = I_{11}^{K_0^*\pi} g_{\mu\nu}$. Diagrams with intermediate axial vector and pseudoscalar mesons in ground and excited states are given by following amplitude:

$$\begin{aligned}
&\mathcal{M}_{K_1+K_1'}{}_{\mu\nu} = \\
&\frac{C_{K_1}}{g_{K_1}} I_{11}^{K_1 K_0^* \pi} \Big[g_{\mu\nu} p^2 f(p^2) - p_\mu p_\nu f(M^2_{K_1(1270)})\Big] BW_{K_1(1270)} \sin^2(\alpha) \\
&g_{\mu\nu} p^2 f(p^2) - p_\mu p_\nu f(M^2_{K_1(1400)})\Big] BW_{K_1(1400)} \cos^2(\alpha)\Big] \\
&+\frac{C_{K_1'}}{g_{K_1}} I_{11}^{K_1' K_0^* \pi} \Big[g_{\mu\nu} p^2 f(p^2) - p_\mu p_\nu f(M^2_{K_1(1650)})\Big] BW_{K_1(1650)}, \quad (17)
\end{aligned}$$

$$\mathcal{M}_{K+K'\mu\nu} = 4m_sF_K\Big[I_{11}^{KK_0^*\pi}BW_K + \gamma_{K'}I_{11}^{K'K_0^*\pi}BW_{K'}\Big]g_{\mu\nu}p_\nu, \tag{18}$$

where $f(p^2) = 1 - 3(m_s + m_u)^2/2p^2$, $\gamma_{K'} = F_{K'}/F_K \approx 0.26$ [12]; $C_{K_1} = 0.90$ and $C_{K_1'} = 0.42$ [12]; $p = p_{K_0^*} + p_\pi$. The integrals over the quark loop are defined in (19). Intermediate mesons are described by Breit-Wigner propogators

$$BW_M = \frac{1}{M_M^2 - p^2 - I\sqrt{p^2}\Gamma_M}, \tag{19}$$

where masses and widths of mesons are taken from PGD [19].

For the decay $\tau \to K_0^*(1430)\pi\nu_\tau$ the amplitude has a similar structure with the replacement of the vertices $WK_0^*(800)\pi \to WK_0^*(1430)\pi$, $K_1K_0^*(800)\pi \to K_1'K_0^*(1430)\pi$ and meson masses $M_{K_0^*(800)} \to M_{K_0^*(1430)}$. Next consider the decay of $\tau \to K_0^*(800)K\pi\nu_\tau$. This decay differs from the decays described above with presence of channels a_1, a_1', π and π' states [23]

$$\begin{aligned}
&\mathcal{M}(\tau \to K_0^*(700)K\nu_\tau) = \\
&2\sqrt{2}G_FV_{ud}L_\mu\left[\frac{C_{a_1}}{g_{a_1}}\Big[g_{\mu\nu}(p^2 - 6m_u^2) - p_\mu p_\nu\Big]BW_{a_1}I_{11}^{a_1K_0^*K}\right. \\
&+\frac{C_{a_1'}}{g_{a_1}}\Big[g_{\mu\nu}\left(p^2 - 6m_u^2\right) - p_\mu p_\nu\Big]BW_{a_1'}I_{11}^{a_1'K_0^*K} \\
&\left.+4m_sF_\pi\left(I_{11}^{\pi K_0^*K}BW_\pi + \gamma_{\pi'}I_{11}^{\pi'K_0^*K}BW_{\pi'}\right)g_{\mu\nu}p_\nu\right]\left(p_{K_0^*} - p_K\right)_\nu,
\end{aligned} \tag{20}$$

where $\gamma_{\pi'} = F_{\pi'}/F_\pi \approx 0.054$ [12] and $p = p_{K_0^*} + p_K$.

The amplitude of the decay $\tau \to K_0^*\eta\nu_\tau$ takes the form

$$\begin{aligned}
&\mathcal{M}(\tau \to K_0^*(700)\eta\nu_\tau) = \\
&8G_FV_{us}F_KL_\mu\left[\left(m_sI_{11}^{KK_0^*\eta_u} - \sqrt{2}m_uI_{11}^{KK_0^*\eta_s}\right)BW_K\right. \\
&\left.+\gamma_{K'}\left(m_sI_{11}^{K'K_0^*\eta_u} - \sqrt{2}m_uI_{11}^{K'K_0^*\eta_s}\right)BW_{K'}\right]\left(p_{K_0^*} + p_\eta\right)_\mu,
\end{aligned} \tag{21}$$

here we take into account the u, d and s quark parts of the η meson.

Branching fractions for decays with strange scalar mesons within the extended NJL model are [23]

$$\begin{aligned}
Br(\tau \to K_0^*(800)\pi\nu_\tau)_{NJL} &= 4.75 \times 10^{-4}, \\
Br(\tau \to K_0^*(1430)\pi\nu_\tau)_{NJL} &= 1.80 \times 10^{-6}, \\
Br(\tau \to K_0^*(800)K\nu_\tau)_{NJL} &= 5.00 \times 10^{-4}, \\
Br(\tau \to K_0^*(800)\eta\nu_\tau)_{NJL} &= 3.59 \times 10^{-8}.
\end{aligned} \tag{22}$$

4 Conclusion

In this paper, we present the fundamentals of deriving the Larganjian for strange scalar mesons and calculate the mass spectrum. Calculations within the NJL model confirm the quark-antiquark structure of mesons $K_0^*(800)$ and $K_0^*(1430)$. Theoretical estimates obtained within the model for strong meson decays are in agreement with recent BaBar data [18]. Comparisons of the coupling constants for strong decays of the radially excited meson $K_0^*(1430) \to K\pi$, $K_0^*(1430) \to K\eta$ and $K_0^*(1430) \to K\eta'$ are also presented. The calculated partial widths of τ lepton decays into scalar-pseudoscalar meson pairs are presented in Section 3. The contribution from axial-vector channels dominates in determining the decay widths. The states $K_1(1270)$, $K_1(1400)$, and the excited one $K_1(1650)$ act as axial-vector intermediate mesons in the decays $\tau^- \to K_0^*(800)^-\pi^0\nu_\tau$ and $\tau^- \to K_0^*(1430)^-\pi^0\nu_\tau$. The decay $\tau^- \to K_0^*(800)^- K\nu_\tau$ occurs with the contribution of channels with non-strange states a_1, a_1', π, and π'. Processes involving strange scalar mesons have not been well studied experimentally. The measurements of heavy meson decays indicate the importance of the contributions of intermediate channels with the $K_0^*(1430)$ state [18, 24, 25, 26]. The results of the present work on τ decays are a prediction for future experiments on Super $c - \tau$ factories [27].

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